suggests a method of determining the line length without reentering the chart and uses a construction similar to that proposed by Arnold [1].

We will consider the example of transforming $Z_L = (1 + j1)$ to $Z_S = (2 + j3)$. A suitable normalizing impedance is 2 Ω since this ensures that Somlo's circle through the two points is centered near the chart origin, the condition for greatest accuracy occuring when it is exactly centered. Somlo's construction yields a circle centered at C and with intercepts at 0.45 and 5.0; whence $Z_0 = 1.5$, this point is entered on the chart (Fig. 1). This construction has been left out for clarity but the chart center is marked; we note that this point only coincides with Z_0 either when both lie on the origin or the circle has zero radius. Construct a circle passing through Z_L and the ends of the real axis A and A'. Extend the line $Z_L - Z_0$ to the circle at P, then construct $P - 0 - l_L, l_L$ is then the angle equivalent of the reflection coefficient for Z_L. A similar construction for Z_S yields l_S and the length of transmission line required is the difference between the two values of l, the direction of rotation is as in normal Smith chart practice and $dl = 0.086\lambda$.

Use of the Smith chart in the arbitrary normalization mode enables one to define the range of impedances to which a known impedance may be transformed using a single matching section. Using $Z_L = (1 + j1)$ normalized to 2 Ω we enter the chart in Fig. 2. The impedance can be matched to any other provided the circle constructed through the two impedance points does not intercept the chart boundary at $\rho = 1$. This limits the range of impedances to which Z_L may be matched to the area shaded on Fig. 2.

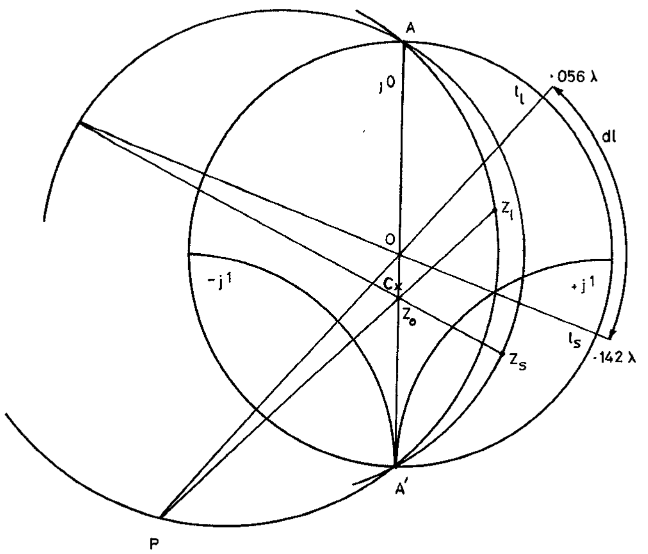

Fig. 1. Construction for transforming $Z_L = (1 + j1)$ to $Z_S = (2 + j3)$.

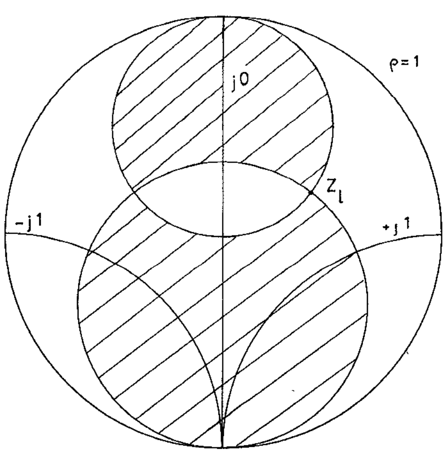

Fig. 2. Area of simple matching impedances to $Z_L = (1 + j1)$.

ACKNOWLEDGMENT

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Analysis of DC Blocks Using Coupled Lines

CHEN Y. HO

***Abstract*—It is shown mathematically that dc blocks can be realized by using λ/4 − 3-dB directional couplers with both coupled port and transmitted port open-circuited.**

Using λ/4 − coupled-line structure, dc blocks in microwave frequency have been realized in the past [1]. The approach of analysis used in [1] employs an approximate equivalent circuit based on the even- and odd-mode propagation in coupled lines of [2]. It is the purpose of this letter to show that this type of dc block can be analyzed exactly as a 3-dB directional coupler with both coupled port and transmitted port open-curcuited. The results are general and can be applied to any type of realization of 3-dB directional couplers, not necessarily restricted to microstrip edge-coupled lines.

The schemetic of conventional directional couplers is shown in Fig. 1 in which port 1 is assumed the input port, port 2 is the coupled port, port 3 is the isolated port, and port 4 is the transmitted port.

By assuming that ports 1 and 3 are terminated with the characteristic impedance Z_0, while ports 2 and 4 are open-circuited, the impedance matrix of this four-port network [3] can be simplified as follows:

$$\begin{bmatrix} V_1 \\ V_3 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{13} \\ Z_{31} & Z_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_3 \end{bmatrix} \tag{1}$$

where V_1, I_1, V_3, and I_3 are voltages and currents at port 1 and port 3, respectively, and

$$Z_{11} = Z_{33} = \frac{Z_{0e} + Z_{0o}}{2s} = \frac{1}{s(1 - k^2)^{1/2}} Z_0 \tag{2}$$

$$Z_{13} = Z_{31} = \frac{(Z_{0e} - Z_{0o})\ (1 - s^2)^{1/2}}{2s} = \frac{k(1 - s^2)^{1/2}}{s(1 - k^2)^{1/2}} Z_0 \tag{3}$$

where

Z_{0e} even-mode impedance of the coupled lines;
Z_{0o} odd-mode impedance of the coupled lines, and $Z_{0e}Z_{0o} = Z_0^2$;
k voltage coupling coefficient;
s $(-1)^{1/2} \tan(\theta)$, θ is the electrical length.

After substituting the relation $V_3 = -Z_0 I_3$ into (1), and eliminating the variable I_3, we obtain

$$\frac{V_1}{I_1} = Z_{in1} = Z_{11} - \frac{Z_{13}Z_{31}}{Z_0 + Z_{33}} \tag{4}$$

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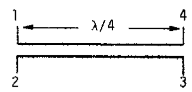

Fig. 1. Conventional directional coupler.

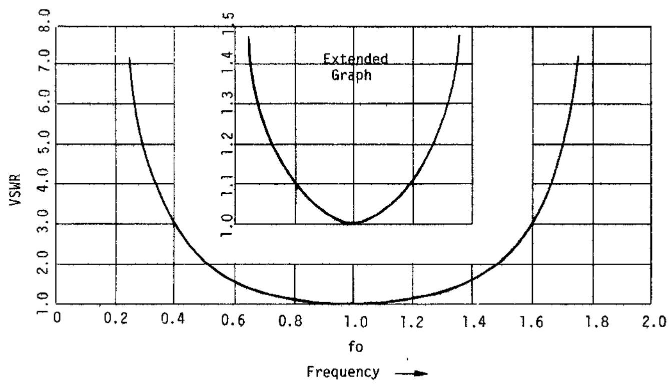

Fig. 2. Input VSWR versus normalized frequency.

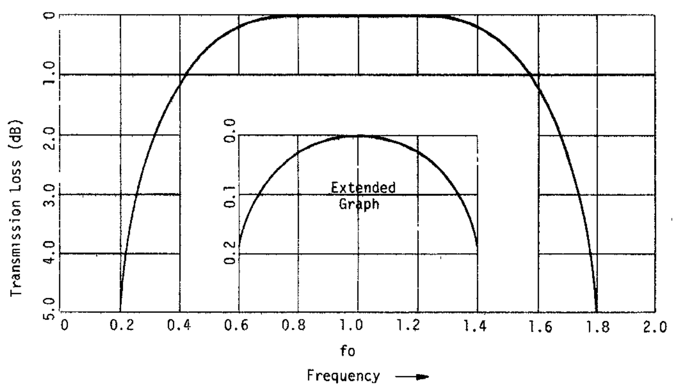

Fig. 3. Transmission loss versus normalized frequency.

or

$$Z_{\text{in1}} = \frac{s^2 + (2)^{1/2} s + 1}{s^2 + (2)^{1/2} s} Z_0 \tag{5}$$

where $k = 1/(2)^{1/2}$ has been used, and Z_{in1} is the input impedance at port 1.

The input VSWR at port 1 can be calculated using (5) as

$$VSWR = \frac{|Z_{\text{in1}} + Z_0| + |Z_{\text{in1}} - Z_0|}{|Z_{\text{in1}} + Z_0| - |Z_{\text{in1}} - Z_0|} = 1/\sin^2(\theta). \tag{6}$$

The transmission loss from port 1 to port 3 can be calculated using (5) as

$$T = 20 \log_{10}\left(1 - \left(\frac{|Z_{\text{in1}} - Z_0|}{|Z_{\text{in1}} + Z_0|}\right)^2\right)^{1/2} = 20 \log_{10}\left(\frac{2 \sin(\theta)}{1 + \sin^2(\theta)}\right) \text{ (in dB)}. \tag{7}$$

Equations (6) and (7) are plotted and shown in Figs. 2 and 3, respectively.

Note that Fig. 2 is slightly different from [1, fig. 2]. The differences are due to the different coupling coefficient and the approximate equivalent circuit used in [1].

The other particular case where both port 2 and port 4 are shorted to ground can be analyzed in a similar manner provided that the admittance matrix instead of impedance matrix be used. The input VSWR and the transmission loss can be shown identical to (6) and (7), respectively.

We conclude that dc blocks in microwave frequency can be realized by using $\lambda/4 - 3$-dB directional couplers with both coupled port and transmitted port open-circuited. The exact theoretical responses of input VSWR and transmission loss have been derived.

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The Synthesis of Quarter-Wave Transformers, Low-Pass and Half-Wave Filters in the Sine Plane

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***Abstract*—This letter presents a method for synthesizing quarter-wave Chebyshev transformers, low-pass and half-wave filters. The method uses sine-plane synthesis which greatly simplifies the numerical calculations and allows one to obtain good numerical accuracy using just a desk calculator. Equations for the transmission coefficient are given in a simplified form.**

With the discovery of the S-plane equivalent circuit of the transmission line and the development of S-plane synthesis techniques [1], [2], it is possible to synthesize quarter-wavelength Chebyshev transformers and low-pass filters with a high degree of accuracy using a desk calculator. This letter presents the equations for the quarter-wave transformer in its simplest form and demonstrates by examples the synthesis procedure for both transformers and low-pass filters.

The equations for the quarter-wavelength transformer have been derived by a number of authors [3]–[5]. The method presented here follows [5] which is believed to be the simplest form. The desired transmission coefficient is given by

$$T = \frac{1}{1 + k^2 T_n^2(x)} \tag{1}$$

where T_n is the nth order Chebyshev polynomial and

$$x = \frac{\cosh \bar{\theta}}{p} \tag{2a}$$

$$p = \cos \theta_0. \tag{2b}$$

Then

$$x^2 = \frac{1}{p^2(1 - \lambda^2)} \tag{2c}$$

where

$$\lambda = \tanh \bar{\theta}. \tag{2d}$$

$\bar{\theta}$ is a complex angle which on the j axis equals $j\theta$ and x then is equal to $\cos \theta / \cos \theta_0$ where θ_0 and $\pi - \theta_0$ are the cutoff angles of the transformer. The transformer circuit is shown in Fig. 1 and is normalized

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